



LAKE SUPERIOR
STATE UNIVERSITY

OFFICE OF THE PROVOST

APPLICATION FOR SABBATICAL LEAVE
(Refer to Section 15.4 of the Faculty Association Agreement)

I. Name Brian Snyder Date November 2, 2010
Department Mathematics and Computer Science Ext. No. 2658
Home Address [REDACTED] Home Phone [REDACTED]

II. Application for leave during the following (*indicate semester and/or year*):

Fall _____ Spring _____ Full Year AY 2011-12

III. Number of years of faculty service (*minimum of 5 years required*) 11

IV. Tenure status (*tenure required*) Tenure awarded 9/2005

V. Semester or year of last sabbatical (*if applicable*) Not Applicable
(*minimum of 5 years since last sabbatical required*)

VI. Title and description of sabbatical project (*attach pages as appropriate*). Include in the description a discussion of at least one of the following criteria:

1. The strength of the relationship between a sabbatical leave proposal involving applied or theoretical research related to professional activities and the advancement of knowledge within disciplinary areas.
2. The strength of the relationship between a sabbatical leave proposal involving an external, professionally-related experience/study in a business, industrial, health care, scientific or educational setting and the improvement of instructional/professional activities at the University.
3. The strength of the relationship between a sabbatical leave proposal involving travel or advanced study and its yield in improving the quality of instruction at the University.

VII. Attach a statement agreeing to return to the University.

Please return to the Office of the Provost.

Mathematics Visualization Program (MVP) – Developing Visual Media in the Teaching and Learning of Mathematics

Abstract

I am applying for a year-long sabbatical to develop instructional materials and new pedagogical methods in the teaching of mathematics. Work performed during the sabbatical period is designed to contribute to two general projects, which are detailed below. To complete these projects, I will be spending time performing research in an interdisciplinary area that is under-represented, learning state-of-the-art software applications and combining the products in new ways, developing opportunities to practice my craft as a teacher, and producing tangible assets that will enhance the reputation of Lake Superior State University.

Project 1: Complete textbook on Art of Mathematics and develop supplementary materials.

I have taught a course for the Honors Program at LSSU entitled “The Art of Mathematics” twice, during the Spring 2003 and Spring 2005 semesters. The guiding theme of the course is to show that mathematics has an intrinsic aesthetic appeal, that art has been influenced by mathematics (geometry in particular) throughout history, that symmetry in nature has a mathematical basis and that art and mathematics are trying to answer the same questions about our world, albeit in different ways. A copy of the syllabus is attached to this narrative.

This course was brought before the General Education committee in Fall 2002 and was approved to satisfy the mathematics requirement. The philosophy of Art of Mathematics is that anyone with some basic algebraic skills and a willingness to explore can find these applications of mathematics beautiful, accessible and enjoyable. While this course was listed as an honors course, it was open to any interested student who met the mathematical proficiency requirement (MA 086). The design of the course was to start at an approximate level of the minimum general education course MATH 110– Explorations in Mathematics. Some concepts which normally appear in upper level mathematics courses, such as symmetry groups and duality, are explored in a very accessible manner.

Since a course can only be offered twice as a special topics course before it must be sent to the Curriculum Committee to be considered as a regular course offering, I have been holding on to the material that I gathered while preparing and teaching these classes and plan to compile these diverse sources into a single text. Additionally, several of my colleagues have suggested that I compile this material into a book and would be very interested in reading it. In addition, since that time I have been collecting additional material, including additional references books and articles, personal photographs in China, Italy, Chile, Argentina, Greece and Turkey; studies in ornament and design and the cultural significance thereof; and retrospectives and technical analyses of the work of Dutch graphic artist M. C. Escher.

The current course consists of a progression of geometric ideas as they were discovered in chronological order, beginning with classical Euclidean (plane) geometry,

followed by projective geometry from the Renaissance, then an introduction to group theory, which is the language of symmetry, finishing up with a brief entry in fractal geometry. Both times the course was offered an exhibition of artistic work created by the class, from the students as well as the instructor, has been shown in the Library gallery. Since these have been term projects for the Spring semester, the library staff asked both times to allow us to keep the exhibition over the summer, which we were happy to do.

In addition to the original course material, I would like to expand a couple of the topics and develop a new area to the original course material. Currently, my material is a combination of different electronic formats and a collection of physical artifacts. These need to be collected into a single format and location. As a complement to plane geometry I would like to develop teaching material in non-Euclidean geometry. Second, I would like to create content that extends the usual work in projective geometry. Currently, most published work involves using projective geometry to accurately portray a realistic object. Applications of non-accurate use of projective geometry are optical illusions and impossible objects. Finally, I would like to create supplemental material to go along with the main text. This material is best explored actively. Having a collection of different worksheets, small projects and interactive computer demonstrations to complement the main text will allow me to address a broad audience, from instructors looking to teach a similar class to individuals hoping to learn the material on their own.

Project 2: Develop instructional materials for my text Algebraic Geometry: A Problem Solving Book to provide professional development opportunities for mathematicians.

During the Spring 2008 semester I was accepted to participate in the Park City Mathematics Institute (PCMI) Summer Program in Park City, Utah. Funding for my participation for this 3-week residential program was provided by the Institute for Advanced Study affiliated with Princeton University in Princeton, New Jersey. My primary focus was the Undergraduate Faculty Program, intended for faculty who teach at primarily undergraduate institutions, which was led by Professor Tom Garrity of Williams College in Williamstown, Massachusetts. During the course of the Program most of the participants (11 out of 13) decided to collaborate on the writing of a textbook in Algebraic Geometry, which was the main focus of the Summer Program. During our time together we created a table of contents and began to complete the text, including problems and solutions. Before the end of the Summer Program, I volunteered to serve as the technical editor in addition to my regular writing contribution. This text has evolved into Algebraic Geometry: A Problem Solving Approach.

During the Fall 2008 semester we primarily worked on the text using a combination of e-mail and a version control program to allow multiple authors to work on a project. A class was taught using this text as the primary source during the Spring of 2009 at the Georgia College and State University, the home institution of one of the authors. Subsets of the authors have met face-to-face at national meetings as a matter of convenience; the authors' home institutions are located from Maine to Georgia in the eastern United States to Texas and Colorado in the west. We have since met twice; once in January 2009 in Washington, D.C. and in August 2010 in Pittsburgh, Pennsylvania.

As of this writing, the text is nearly complete; a copy of the table of contents is attached to this narrative. The vast majority of items remaining to be done are static illustrations. A preliminary version of the text has been submitted to a senior acquisitions editor from the American Mathematical Society and the initial review was very positive and encouraging.

The next step in the process is developing material that can be used in workshops and short courses for mathematics faculty who would like to learn more about algebraic geometry. These workshops will be led by members of the authoring group, including myself. Short courses, which are four to eight hours long, are offered in conjunction with the major national meetings of the mathematical community, the Joint Mathematics Meetings held in January, and the Mathematical Association of America's MathFest held in August. The next national meetings are scheduled for Lexington, Kentucky (August 2011), Boston, Massachusetts (January 2012), Madison, Wisconsin (August 2012) and San Diego, California (January 2013). In addition, this may be offered as a workshop, which can last from one day to one week in duration. The MAA has a competitive funding program through their Professional Enhancement Programs (PREP) workshops. If funded, the MAA will help defray the housing and subsistence costs for participants and provide advertising for the program. The application deadline is expected to be July 31, 2011 for the Summer 2012 PREP workshop.

Technical Details

The standard for mathematical typesetting is LaTeX. Working together in Summer 2008, the authoring group developed style, labeling and content guidelines. During the writing of the algebraic geometry textbook I enhanced my skills in creating my portion of the LaTeX documents. Additionally, as the technical editor I was responsible for ensuring that the co-authors were consistent in following the guidelines. A couple of authors have been creating illustrations embedded within LaTeX using a specialized package called pstricks that creates PostScript graphics. As technical editor, I have had to learn this package to be able to edit the illustrations. These skills can be transferred to the Art of Mathematics text.

There are additional software packages that I would like to use in completing these projects. Lake Superior State University has a license for the computer algebra system *Mathematica*. I have used *Mathematica* in creating in-class illustrations and more complicated curves than are available in the pstricks macros. This computer algebra system can provide output in many different formats. Another program that I am experimenting with is Maya. This program is used in the entertainment industry to build, render, and animate virtual objects in a 3-dimensional environment. The output can be converted into a still picture or a motion picture in a variety of sizes, resolutions and formats.

I have been using *Mathematica* for 16 years and have begun working with Maya two months ago. Most of what I have learned has been self-taught, designed to solve an immediate problem of my own. I will spend some time learning how to use these programs more efficiently and effectively and be able to have these programs used by the larger community. The maker of *Mathematica* offers classes in advanced programming and visualization. The publisher of Maya has entered into agreements with training

facilities around the world to provide training and certification in the use of their products, including Maya. These courses range from one day to one week in length and range in price from \$300 to \$3,000. As of this writing, courses are scheduled until March 2011. If I receive this sabbatical, I will be able to enroll in courses during the sabbatical period when future courses are announced.

Travel Plans

I will be travelling to complete many of the tasks listed in this narrative. During the early stages I will be concentrating on completing the short course in algebraic geometry and locating additional sources for the Art of Mathematics. These can be combined geographically. The project lead for the algebraic geometry book is at Williams College in western Massachusetts. The nearest airport is located in Albany, New York, which is the home of the editor and major contributor to the journal Mathematics and the Arts. The other current contributing co-authors are located in Milledgeville, Georgia, Hillsdale, Michigan, Springfield, Missouri and Meadville Pennsylvania.

There are several research libraries that I would like to access. As a life member of The Ohio State University Alumni Association, I have borrowing privileges to all university libraries along with limited privileges with the statewide library system OhioLink. As a faculty member at a university in the state of Michigan, I can apply to access the libraries at the University of Michigan, including the special collection of mathematics texts. Furthermore, I hope to access the Dibner Library of the History of Science and Technology of the Smithsonian Institution to review material from their collection of historic mathematics works. There is a fellowship application for scholars in residence at Dibner, which covers some expenses. The fellowship application deadline is expected to be April 30, 2011 for the 2012 calendar year. Even if I am not successful in earning a fellowship, I intend to perform research there, albeit for a shorter period of time.

Proposed Timeline

The major component of the sabbatical is the creation of the written material and the creation of electronic media for both projects. To assist in the creation of the electronic media, I will be spending some time taking classes in Maya and *Mathematica* and learning how to use *Mathematica* to assist creation of 3-dimensional objects in Maya. Travel to museums and libraries for primary source research will be scheduled around the software classes. In addition, I will attend several professional meetings to meet with colleagues. Initially, these meetings will be collaborations with co-authors while later meetings will be committed to teaching the mini-course in Algebraic Geometry. Based on these parameters, I have developed the following schedule of major events. Non-specified time will be spent on the creation of the material and media.

- September 2011

- (1 week) Travel to Williams College – complete Mini-Course proposal and finalize outlines for workshops and courses (4 hour, 8 hour and 20 hour versions)
- (2 days) Travel to University of Michigan to obtain guest researcher privileges, review Special Mathematics Collection items, research Art Museum for illustrative material for Art of Mathematics
- (3 days) Travel to Museum of Arab-American History (Dearborn, MI), Detroit Institute for the Arts and Detroit Science Center for Art of Mathematics
- TBD: Mathematical Association of America Upper Peninsula Regional Meeting – Sault Ste. Marie, MI
- October 2011
 - (1 week) Introduction to Maya – Chicago, IL
 - October 11: Mini-Course Proposal due for MathFest 2012
 - (10 days) Travel to The Ohio State University to perform research at Science and Engineering Library, Special Collections and galleries
 - Announcement of PREP workshop funding
- November 2011
 - (1 week) A First Course in *Mathematica* (M 101)
 - (2 – 3 days) Travel to University of Michigan for research at Special Mathematics Collection
- December 2011
 - (2 days) Visualization in *Mathematica* (M 205)
 - December 27: Mini-Course Proposal due final Tuesday for Joint Mathematics Meetings 2013 (San Diego, CA)
- January 2012
 - January 4 – 7: Joint Mathematics Meetings – Boston, MA Authoring Group work session
 - (1 week) Advanced Maya – Chicago, IL
- February 2012
 - (1 week) Introduction to *Mathematica* Programming (M 221)
- March 2012
 - (one week – one month, pending fellowship) Travel to Dibner Library of Science and Technology and affiliated Smithsonian Libraries and Museums to perform research on original sources for Art of Mathematics
- April 2012
- May 2012
 - TBD: Michigan MAA Meeting – University Center, MI
- June 2012
 - TBD: Advanced Placement Calculus Exam Grading – Kansas City, MO
- July 2012
 - TBD: PREP workshop – Location to be negotiated with co-authors
- August 2012
 - August 2 – 4: MathFest – Madison, WI Teach Mini-Course in Algebraic Geometry

November 9, 2010

To Whom It May Concern:

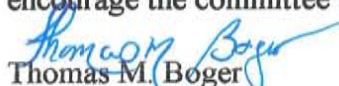
When I learned that Dr. Brian Snyder planned to apply for a sabbatical leave for the 2011-12 academic year, I offered to write a letter expressing my support for his application. Having a departmental faculty member on leave will certainly present challenges, but there's no question in my mind that the potential advantages far outweigh those short-term difficulties.

During the time that Brian has been at Lake Superior State University, classroom responsibilities have, unfortunately, pulled him away from the research areas that were the focus of his graduate education. Because of the diversity of course offerings within our area, he's had a minimum of three preparations, and more often four preparations, every semester. After ten years, it would be difficult for Brian to reconnect with that work, but he has identified two other projects that he hopes to complete during his sabbatical.

I don't know that I've ever seen Brian more enthusiastic, or his students more engaged, than during the semesters when he presented the "Art of Mathematics" course. As he notes in his application, this course was initially developed for the LSSU Honors program, although most of the material should be accessible to students with relatively modest mathematical foundations. If this course was to be approved as a regular offering in Mathematics, it could continue to serve the Honors program, but I think it also has great potential as a source of enrichment content for both elementary and secondary mathematics teachers. It may also be possible to re-package this content as a continuing education experience for area teachers. This sabbatical would definitely provide Brian with the opportunity to pull together the materials necessary to make this happen.

The second project that he's identified is perhaps not as immediately relevant to the LSSU classroom, although I believe our stronger students in mathematics would find the content useful in their preparations for graduate study. More importantly, the material he's working on, and would like to finish up during the sabbatical, is of great value to the larger mathematics community. Successfully bringing this project to a conclusion would definitely raise Brian's profile, and, by extension, the profile of Lake Superior State University, within the mathematics community.

During his tenure at LSSU, Dr. Snyder has shown a great deal of intellectual curiosity, and has devoted a great deal of effort to the integration of technology into the mathematics classroom. The projects he has outlined promise to push Brian's activities in this area to a higher level, and could be a real benefit to his colleagues in mathematics. I strongly support his application, and encourage the committee to do so, as well.



Thomas M. Boger
Chair, Mathematics and Computer Science

thomas.garrity@williams.edu

November 6, 2010

To Whom It May Concern

I strongly support Brian Snyder's proposal for a sabbatical. Brian has the potential for becoming a national leader for getting mathematicians involved in innovative scholarship. I first started working with Brian in the summer of 2008 at the three week Park City Mathematical Institute session on Complex Algebraic and Analytic Geometry. Algebraic Geometry has been at the core of modern mathematics for at least the last two hundred years, if not from the time of Descartes and his analytic geometry. I was in charge of the Undergraduate Faculty Program, which had as its goal to introduce faculty from primarily undergraduate institutions to algebraic geometry and how to teach this subject to undergraduates. About twelve of us, including Brian, decided to take these beginning notes and transform them into an introductory text in algebraic geometry. This text will be quite different than any existing one as it is based on many, many problems, which will allow the reader to build up the needed insights and intuitions for truly understanding this difficult area of math. There are three intended audiences: students who have just finished a linear algebra class, upper level students and beginning graduate students and finally professional mathematicians who want to learn some algebraic geometry. We hope that the book will be a resource for many years to come.

This book's existence is a model for how other mathematicians, especially for those who are not working at research institutions, can set up collaborations for scholarship. I have long felt that there is a vast untapped reservoir of untapped mathematical talent in this nation. Many, if not most, mathematicians leave graduate school and head off to schools whose primary goal is teaching. These folks will devote the bulk of their professional lives to the teaching of their students. This is good. It does have the consequence, though, of hindering any attempt for a serious research program. This book has twelve

authors. All of the authors are at schools with substantial teaching obligations (save for possibly my school, where we have a somewhat light teaching "load"): for the same reason, all of the authors are at schools without major research expectations (again, save for my school). This book is creating for us a scholarly community. Further, since the authors care about and are good at teaching, the exposition will hopefully be quite good. We are hoping that this multi-author text will become a model for many other mathematicians, benefiting not only these future authors but the entire mathematical community. Imagine these scholarly networks, spread across the nation, which keep the individual professor mathematical alive and creative while at the same time producing excellent works that others can actually use and learn from.

Brian is one of the key players in this collaboration. As of now, he is the primary person keeping the project coordinated. Even more importantly, he is a great writer. In the last few years there have been times when we were in daily contact for a number of weeks. I learned a lot from him. The book will owe much of its eventual success to Brian.

We are planning to set up various workshops for other mathematicians to learn to teach from the book (there are mechanisms for this in both the American Mathematical Society and the Mathematical Association of America, our two main professional societies) and for how to set-up these scholarly networks. This planning will take time, effort and creativity. Brian will use his sabbatical not only to finish the book, (which to be honest will be finished whether or not he gets this sabbatical) but more importantly to start this planning.

Thus Lake Superior State University will benefit greatly from granting Brian a sabbatical.

Sincerely,

A handwritten signature in blue ink that reads "Thomas Garrity".

Thomas Garrity
William R. Kenan Jr. Professor of Mathematics
Director: Williams College Project for Effective Teaching

HP 202 – Art and Mathematics

Spring 2005

Prerequisite: MA 086 with a grade of C or higher and permission of Honors Program Director.

Instructor: Dr. B. Snyder

Office: CAS 206-A1

Phone number: (906) 635-2658 (x2658 on campus)

e-mail address: bsnyder@lssu.edu

Office Hours: MT 2:00-2:50, W 9:00-9:50, RF 11:00-11:50 and by appointment.

Course URL: <http://math.lssu.edu/bsnyder/S05/HP202/>

Texts:

- Gardner, Martin Penrose Tiles to Trapdoor Ciphers . . . and the Return of Dr. Matrix ISBN 0-88385-521-6 (Paperback) Mathematical Association of America
- Lawlor, Robert Sacred Geometry ISBN 0-500-81030-3 (Paperback) Thames & Hudson 2001
- Lundy, Miranda Sacred Geometry ISBN 0-8027-1382-3 Wooden Books 2001
- Pedoe, Dan Geometry and the Visual Arts ISBN 0-486-24458-X (Paperback) Dover Books 1983
- Schneider, Michael S. A Beginners Guide to Constructing the Universe: the Mathematical Archetypes of Nature, Science and Art ISBN 0-06-092671-6 (Paperback) Harper Perennial 1994
- Weyl, Hermann Symmetry ISBN 0-691-02374-3 (Paperback) Princeton University Press 1983
- Frantz, Marc Lessons in Mathematics and Art (available on-line at <http://php.indiana.edu/~mathart/viewpoints/lessons/>)

Course Tools: You will need to purchase a compass and straightedge (or combination of the two) and a bound notebook, preferably with unlined paper. You will also need access to art supplies throughout the term. We will also be using some non-traditional art supplies, such as masking tape, chopsticks, string and access to Microsoft Excel.

Course theme: This course is intended to show that mathematics has an intrinsic aesthetic appeal, that art has been influenced by mathematics (geometry in particular) throughout history, that symmetry in nature has a mathematical basis and that art and mathematics are trying to answer the same questions about our world, albeit in different ways. We will be examining different geometries (classical, projective and fractal) and seeing how these are represented in the art of the period. We will be looking for symmetry in art and nature and developing a mathematical way of expressing these symmetries. We will look for common themes in art from different periods. Class time will consist of discussions, along with individual and group projects. We will be forming a community to explore new ideas and develop insights to share with others. Attendance at every class session will allow you to get the most out of the course. This course will count towards the Mathematics component of the General Education Curriculum at Lake Superior State University.

Grades: The final grade in this course will be based on the scores that you will earn on the examinations that you will take in this class and your participation during the course. There will be two 100-point exams during the term and a 150 point final examination given during the final exam week. **Make-up exams will not be given without instructor approval and documentation.** Homework will be assigned on a regular basis. This will consist of exercises from the texts, written responses to material presented in class and personal observations made outside of class. These will be completed in your notebook and submitted on a regular basis. These exercises will be worth a total of 150 points. A term project is required. You will have to create a mathematically based work of art. This collection will be displayed in the art gallery in the Shouldice Library during finals week. This project will be worth 150 points. Attendance and participation will be worth 50 points. The points earned will be totaled out of 700 possible points and final grades will be assigned according to the following scale:

Percentage	Grade
93-100	A
90-92	A-
88-89	B+
83-87	B
80-82	B-
78-79	C+
73-77	C
70-72	C-
68-69	D+
60-67	D
0-59	F

Course Schedule: Since the bookstore does not have all of the texts available, topics and assignments will be distributed on a weekly basis. A course outline with reading, writing and mathematical assignments will be handed out as soon as possible.

In compliance with Lake Superior State University policy and equal access laws, disability-related accommodations or services are available. Students who desire such services are to meet with the professor in a timely manner, preferably the first week of class, to discuss their disability-related needs. Students will not receive services until they register with the Resource Center for Students with Disabilities (RCSD). Proper registration will enable the RCSD to verify the disability and determine reasonable academic accommodations. RCSD is located in South Hall office 206, extension 2355.

**Algebraic Geometry: A Problem Solving
Approach**

Park City Mathematics Institute - 2008

Thomas A. Garrity, Project Lead

Richard Belshoff

Lynette Boos

Ryan Brown

Jim Drouihlet

Carl Lienert

David Murphy

Junalyn Navarra-Madsen

Pedro Poitevin

Shawn Robinson

Brian A. Snyder

Caryn Werner

WILLIAMS COLLEGE

MISSOURI STATE UNIVERSITY

TRINITY COLLEGE

Current address: Providence College

GEORGIA COLLEGE AND STATE UNIVERSITY

MINNESOTA STATE UNIVERSITY

Current address: Jim passed away January 23, 2009 during writing of this book.

FORT LEWIS COLLEGE

HILLSDALE COLLEGE

TEXAS WOMAN'S UNIVERSITY

SALEM STATE COLLEGE

UNIVERSITY OF MAINE, PRESQUE ISLE

LAKE SUPERIOR STATE UNIVERSITY

ALLEGHENY COLLEGE

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Preface

0.1. Algebraic geometry

As the name suggests, algebraic geometry is the linking of algebra to geometry. For example, the circle, a geometric object, can also be described as the points

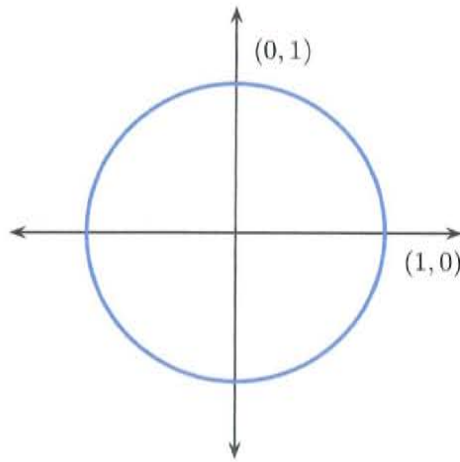


FIGURE 1. The unit circle centered at the origin

(x, y) in the plane satisfying the polynomial

$$x^2 + y^2 - 1 = 0,$$

an algebraic object. Algebraic geometry is thus often described as the study of those geometric objects that can be described by polynomials. Ideally, we want a complete correspondence between the geometry and the algebra, allowing intuitions from one to shape and influence the other.

The building up of this correspondence is at the heart of much of mathematics for the last few hundred years. It touches area after area of mathematics. By now, despite the humble beginnings of the circle

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 - 1 = 0\},$$

algebraic geometry is not an easy area to break into.

Hence this book.

0.2. Overview

Algebraic geometry is amazingly useful, yet much of its development has been guided by aesthetic considerations. Some of the key historical developments in the subject were the result of an impulse to achieve a strong internal sense of beauty.

One way of doing mathematics is to ask bold questions about concepts you are interested in studying. Usually this leads to fairly complicated answers having many special cases. An important advantage of this approach is that the questions are natural and easy to understand. A disadvantage is that the proofs are hard to follow and often involve clever tricks, the origins of which are very hard to see.

A second approach is to spend time carefully defining the basic terms, with the aim that the eventual theorems and their proofs are straightforward. Here the difficulty is in understanding how the definitions, which often initially seem somewhat arbitrary, ever came to be. The payoff is that the deep theorems are more natural, their insights more accessible, and the theory is more aesthetically pleasing. It is this second approach that has prevailed in much of the development of algebraic geometry.

This second approach is linked to solving *equivalence problems*. By an equivalence problem, we mean the problem of determining, within a certain mathematical context, when two mathematical objects are *the same*. What is meant by *the same* differs from one mathematical context to another. In fact, one way to classify different branches of mathematics is to identify their equivalence problems.

A branch of mathematics is closed if its equivalence problems can be easily solved. Currently rich and active branches of mathematics are frequently where there are partial but not complete solutions. The branches of mathematics that will only be active in the future are those for which there is currently no hint for solving any type of equivalence problem.

To solve an equivalence problem, or at least to set up the language for a solution, frequently involves understanding the functions defined on an object. Since we will be concerned with the algebra behind geometric objects, we will spend time on correctly defining natural classes of functions on these objects. This in turn will allow us to correctly describe what we will mean by equivalence.

Now for a bit of an overview of this text. In Chapter 1 our motivation will be to find the natural context for being able to state that all conics (all zero loci of second degree polynomials) are the same. The key will be the development of the complex projective plane \mathbb{P}^2 . We will say that two curves in this new space \mathbb{P}^2 are *the same* (we will use the term “isomorphic”) if one curve can be transformed into the other by a projective change of coordinates (which we will define).

Chapter 2 will look at when two cubic curves are the same in \mathbb{P}^2 , meaning again that one curve can be transformed into the other by a projective change of coordinates. Here we will see that there are many different cubics. We will further see that the points on a cubic have incredible structure; technically we will see that the points form an abelian group.

Chapter 3 turns to higher degree curves. From our earlier work, we still think of these curves as “living” in the space \mathbb{P}^2 . The first goal of this chapter is Bézout’s theorem. If we stick to curves in the real plane \mathbb{R}^2 , which would be the naive first place to work, one can prove that a curve that is the zero locus of a polynomial of degree d will intersect another curve of degree e in at most de points. In our claimed more natural space of \mathbb{P}^2 , we will see that these two curves will intersect in exactly de points, with the additional subtlety of needing to also give the correct definition for intersection multiplicity. We will then define on a curve its natural class of functions, which will be called the curve’s *ring of regular functions*.

In Chapter 4 we look at the geometry of more complicated objects than curves in the plane \mathbb{P}^2 . We will be treating the zero loci of collections of polynomials in many variables, and hence looking at geometric objects in \mathbb{C}^n . Here the function theory plays an increasingly important role and the exercises work out how to bring much more of the full force of ring theory to bear on geometry. With this language we will see that there are actually two different but natural equivalence problems: isomorphism and birationality.

Chapter 5 develops the true natural ambient space, complex projective n -space \mathbb{P}^n , and the corresponding ring theory.

Chapter 6 moves up the level of mathematics, providing an introduction to the more abstract, and more powerful, developments in algebraic geometry in the 1950s and 1960s.

0.3. Problem book

This is a book of problems. We envision three possible audiences.

The first audience consists of students who have taken courses in multivariable calculus and linear algebra. The first three chapters are appropriate for a semester long course for these people. If you are in this audience, here is some advice. You are at the stage of your mathematical career of shifting from merely solving homework exercises to proving theorems. While working the problems ask yourself what the big picture is. After working a few problems, close the book and try to think of what is going on. Ideally you would try to write down in your own words the material that you just covered. Most likely the first few times you try this, you will be at a loss for words. This is normal. Use this as an indication that you

are not yet mastering this section. Repeat this process until you can describe the mathematics with confidence and feel ready to lecture to your friends.

The second audience consists of students who have had a course in abstract algebra. Then the whole book is fair game. You are at the stage where you know that much of mathematics is the attempt to prove theorems. The next stage of your mathematical development is in coming up with your own theorems, with the ultimate goal being to become creative mathematicians. This is a long process. We suggest that you follow the advice given in the previous paragraph, with the additional advice being to occasionally ask yourself some of your own questions.

The third audience is what the authors refer to as “mathematicians on an airplane.” Many professional mathematicians would like to know some algebraic geometry, but jumping into an algebraic geometry text can be difficult. For the pro we had the image of them taking this book along on a long flight, with most of the problems just hard enough to be interesting but not so hard so that distractions on the flight will interfere with thinking. It must be emphasized that we do not think of these problems as being easy for student readers.

0.4. History of book

This book, with its many authors, had its start in the summer of 2008 at the Park City Mathematics Institute’s Undergraduate Faculty Program on Algebraic and Analytic Geometry. Tom Garrity led a group of mathematicians on the basics of algebraic geometry, with the goal being for the participants to be able to teach algebraic geometry to undergraduates at their own college or university.

Everyone knows that you cannot learn math by just listening to someone lecture. The only way to learn is by thinking through the math on your own. Thus we decided to try to write a new beginning text on algebraic geometry, based on the reader solving many exercises. This book is the result.

0.5. An aside on notation

Good notation in mathematics is important but can be tricky. It is often the case that the same mathematical object is best described using different notations depending on context. For example, in this book we will sometimes denote a curve by the symbol C , while at other times denote the curve by the symbol $V(P)$ when the curve is the zero locus of the polynomial $P(x, y)$. Both notations are natural and both will be used.

0.6. Thanks

There are going to be many people and organizations for which the authors are grateful. We would like to thank the Institute for Advanced Study and the Park City Mathematics Institute for their support.

The authors would like to thank the students at Georgia College and State University, Hillsdale College and Texas Woman's University who have course-tested this manuscript and provided many great suggestions.

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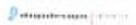
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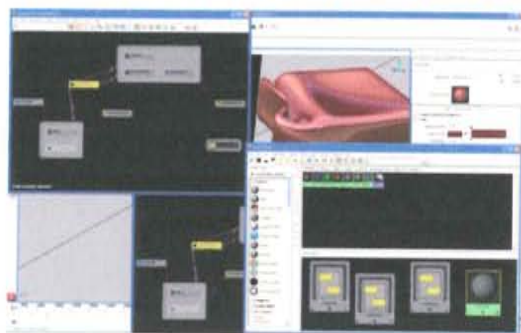
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Introduction to Autodesk Maya

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Course Objectives

Introduction to Autodesk Maya, provides a thorough introduction to Autodesk Maya's Modeling, Texturing, Lighting, and Rendering capabilities. The courseware has been created to help new users make the most of this sophisticated application, as well as broaden the horizons of existing, self-taught users. The material covered in the courseware is designed specifically for professional designers that make up the majority of Autodesk Maya users and includes tutorials designed for professional designers in the Game Development, Architectural, Industrial/Product Design, and Interior Design industries.

Prerequisites

Prior experience using AutoCAD or AutoCAD-based products such as Autodesk Land Desktop, is recommended.

TOPICS INCLUDE:

- User Interface
- New Project Creation
- Polygon Modeling
- NURBS Modeling
- Lighting, Materials, and Shading
- Introduction to Mental Ray in Maya
- UV Texture Editor
- Cameras and Rendering

Instructor Kit includes: Student Guide, instructor notes, and exercise disk.

Price: \$1,750.00

Duration: 4 days

Time: 10am - 6pm

Curriculum: Maya Book
(Price and time may vary outside USA)

Upcoming Class Dates

Click on a date to register.

- [New York](#)
- [Boston](#)
- [Philadelphia](#)
- [Washington DC](#)
- [Orlando](#)
- [Chicago](#)

Dates indicate first day of a multi-day class. Classes usually run over consecutive days.

For additional course dates and availability at our other locations, please [contact us](#).

Related Courses

[Advanced Maya: Lighting, Texturing and Rendering](#)

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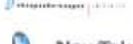
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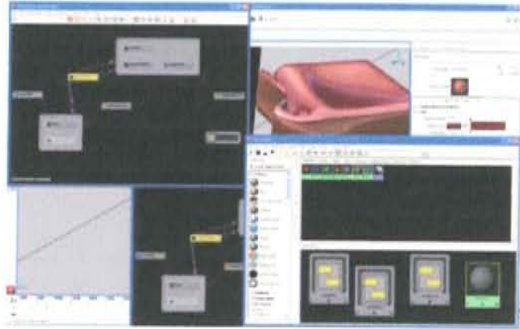
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Advanced Maya: Lighting, Texturing and Rendering

Classes 3D Animation & Design Autodesk Maya Advanced Maya: Lighting, Texturing and Rendering



Price: \$2,495.00
Duration: 5 days
Time: 10am - 6pm
(Price and time may vary outside USA)

Upcoming Class Dates

Click on a date to register.

- New York
- Boston
- Philadelphia
- Washington DC
- Orlando
- Chicago

Dates indicate first day of a multi-day class. Classes usually run over consecutive days. For additional course dates and availability at our other locations, please contact us.

Related Courses

Introduction to Autodesk Maya

Required Prerequisites

[Intro to Maya](#)

Day 1

Topics:

- Product modelling
- Photoreal object modelling techniques for commercials assembling your source material aligning image planes in Photoshop

Day 2

Topics:

- Advanced shading networks
- Layered texture vs. layered shader
- Metal: iron, chrome and copper
- Glass
- Creating wrapping textures

Day 3

Topics:

- Environment textures
- Blending colors
- Facing ratio
- Math utilities
- Render layers review
- Maya vector render engine

Day 4

Topics:

- Intro to mental ray shaders
- Intro to mental ray: ambient occlusion, caustics, global illumination
- HDRI and final gather

Day 5: Using What You Learned

Topics:

- Lighting and texturing your object
- Compositing
- Putting it all together
- Tweaking shadows, specular, reflection and lighting passes
- Creating depth of field
- Outputting the composite

Course Objective

This two-day training course provides direct experience with all the basic features of *Mathematica* as well as a comprehensive foundation for developing advanced applications of the system.

Presenter

The course is presented by a Wolfram Education Group certified instructor.

Target Audience

The course is designed primarily for people who are interested in becoming expert *Mathematica* users but who currently have little or no experience with the system. This course can also be helpful for experienced users who would like to broaden their basic understanding of *Mathematica* and for those interested in learning exactly what the system can do.

Delivery Type

Courses are delivered as instructor-led classes in computer classroom facilities or as online classes over the web. Course topics are presented with alternating sessions of lectures and exercises. All classes feature low student-teacher ratios.

This course is also available in [French](#), [German](#), and [Japanese](#).

Syllabus

This basic course is organized into eight segments.

Introduction

Step-by-step instruction on performing basic operations, building up computations, and navigating the user interface, as well as a description of how to navigate and take full advantage of the documentation system

Programming I

Introduction to the *Mathematica* programming language with emphasis on familiar programming tasks involving procedural, functional, and rule-based styles of programming

Visualization and Graphics

Two- and three-dimensional plotting, plotting data, using options, and creating dynamic and interactive graphics

Working with Notebooks

Introduction to the notebook interface, cells and cell styles, stylesheets, mathematical formulas, hyperlinks and buttons, and slide shows

Symbolic Computation

Computation with symbolic expressions, including polynomial operations, solving equations, functions from calculus, and simplification

Numerical Computation

Fitting data, interpolation, integration, solving equations, displaying intermediate values, differential equations, linear systems, exact vs. inexact numbers, arbitrary-precision numbers, and working with large arrays

Programming II

A deeper look at the syntax and structure of the *Mathematica* programming language, functional programming, pure functions, options and messages, and creating efficient programs

Working with Data

Importing and exporting data and files, file formats, file paths, working with data collections, and visualization of large datasets

Projects

A set of extended projects designed to give practice in using the topics from this course to develop real-world applications

Limited time is frequently available for discussion of special topics chosen by course attendees or at the discretion of the instructor.

Course Materials

Each attendee will be provided with *Mathematica* course notebooks and access to the current version of *Mathematica*. The course notebooks require *Mathematica* or *Mathematica Player*. For attendees participating in classroom-based sessions, course materials are distributed in print and on CD-ROM, and are yours to keep; a computer running *Mathematica* is available for your use during class. For attendees participating in online classes, a download of the course materials is provided; a temporary *Mathematica* training license is provided upon request.

Prerequisites

Course attendees are expected to have experience with common features of modern computer software. Also helpful are knowledge of mathematics through elementary calculus and experience with computer programming at the level of an introductory course in any computer programming language. No prior *Mathematica* experience is required for this course.

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Course Information:

M205: Visualization in Mathematica

DATE & TIME

Jan 11, Jan 13
12 pm - 4 pm EST 

LOCATION

 Online Class

COST

\$225.00 (USD)

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Course Objective

This course provides a foundation for using *Mathematica's* graphical and visualization features as well as working with the graphics programming language. Extensive use of concrete examples and applications to illustrate concepts is included.

Presenter

The course is presented by a Wolfram Research senior developer or a Wolfram Education Group certified instructor.

Target Audience

The course is written for anyone who wants to take advantage of *Mathematica's* graphical and visualization tools, or who has completed "[M101: A First Course in Mathematica](#)."

Delivery Type

Courses are delivered as instructor-led classes in computer classroom facilities or as online classes over the web. Course topics are presented with alternating sessions of lectures and exercises. Additional [online training information](#) is available. All classes feature low student-teacher ratios.

Syllabus

Survey of *Mathematica's* extensive collection of plotting functions

Plotting functions and expressions; visualizing data; working with *Image* data; visualization of networks

Getting the most out of options for visualization functions

Options for color, regions of interest, mesh, vector plot, grid, plot range, text, charting, interactive values, and precedence of arguments

Utilizing graphics programming to enhance your visualizations

The underlying structure of graphics; graphics primitives; graphics directives; *GraphicsComplex*

Techniques for combining plots

Arrays of graphics; merging graphics; using *Inset*; *Prolog* and *Epilog*; *Image* operations

Advanced examples illustrating visualization concepts

Course Materials

Each attendee will be provided with *Mathematica* course notebooks and access to the current version of *Mathematica*. The course notebooks require *Mathematica* or *Mathematica Player*. For attendees participating in classroom-based sessions, course materials are distributed in print and on CD-ROM, and are yours to keep; a computer running *Mathematica* is available for your use during class. For attendees participating in online classes, a download of the course materials is provided; a temporary *Mathematica* training license is provided upon request.

Prerequisites

This course assumes a working knowledge of *Mathematica* syntax and the use of pure functions on the level of "[M101: A First Course in Mathematica](#)."

Course Objective

This one-day course presents an introduction to *Mathematica* programming that enables attendees to develop their own programs to extend *Mathematica*'s capabilities.

Course Summary

This course emphasizes program structure as well as functional and rule-based programming, which is compared to more traditional procedural programming, to help attendees understand and use *Mathematica*'s unique features to their advantage. In the course attendees learn how to solve particular problems more efficiently by choosing the appropriate programming paradigm. The course includes many practical examples and hands-on exercises to help attendees understand the material and to provide a focused and practical learning experience.

Presenter

The course is presented by a Wolfram Education Group certified instructor.

Target Audience

The course is intended for *Mathematica* users who wish to solve problems in their own areas of application and to harness the full power of *Mathematica* by combining its many built-in features in new ways.

Delivery Type

Courses are delivered as instructor-led classes in computer classroom facilities or as online classes over the web. Course topics are presented with alternating sessions of lectures and exercises. All classes feature low student-teacher ratios.

This course is also available in [French](#).

Syllabus

The course is organized into the following segments. Additional topics of interest are covered as time permits. In addition, surveys of further resources are given.

Introduction

An introduction to programming in *Mathematica*; discussion of various programming styles; differences between *Mathematica* and traditional programming languages; structure and syntax of expressions; analyzing expressions

Rules and Patterns

Creating and working with definitions; patterns and pattern matching; conditional patterns; predicates; transformation and replacement rules; the *Mathematica* evaluator

Functional Programming

Functional constructs [Map](#), [Apply](#), and [Thread](#); working with levels in expressions; pure functions; operations on lists; iteration

Mathematica for Procedural Programmers

A presentation of the more traditional programming features such as loops, iterators, scoping and localization, arrays vs. lists, and conditionals

Writing Programs

Discussion of writing larger programs, including default and optional arguments, argument checking, error messages, and usage messages

Optimizing Programs

Example-driven discussion of how to make your *Mathematica* programs as efficient and fast as possible; looping vs. functional approaches; listability; list component assignment; using pure functions vs. traditional definitions; dynamic programming; compiling your programs; efficiency principles

Course Materials

Each attendee will be provided with *Mathematica* course notebooks and access to the current version of *Mathematica*. The course notebooks require *Mathematica* or *Mathematica Player*. For attendees participating in classroom-based sessions, course materials are distributed in print and on CD-ROM, and are yours to keep; a computer running *Mathematica* is available for your use during class. For attendees participating in online classes, a download of the course materials is provided; a temporary *Mathematica* training license is provided upon request.

Prerequisites

Course attendees are expected to have basic familiarity with *Mathematica* approximately equivalent to that provided by "[M101: A First Course in Mathematica](#)." Experience with computer programming at the level of an introductory course in any computer programming language is also helpful.

Sabbatical Report
Fall Semester 2011
Brian Snyder, Ph. D.

Assistant Professor of Mathematics, Lake Superior State University

Synopsis

During the Fall semester of the 2011 – 2012 academic year I was awarded a sabbatical leave to work on a couple of projects that would benefit Lake Superior State University. Thanks to this leave, I was able to complete a textbook, develop skills in animated movie making, assist with the professional development of a mathematics department at a local community college, and obtain materials to begin a couple of new projects.

Itinerary

To complete these activities, I travelled to several institutions of higher learning located in Ann Arbor MI, Meadville PA, Columbus OH and Chicago IL, along with some work performed at LSSU. I visited the Special Collections Rare Book Library at the University of Michigan to review several texts from the 17th and 18th centuries about the state-of-the-art mathematical technology at the time. I travelled twice to work with my co-authors at Allegheny College in Meadville to complete the final draft of our textbook. Allegheny was chosen due to its central location of the group that consisted of collaborators from Hillsdale College, Williams College, and Allegheny College. I was able to return to The Ohio State University as a Visiting Scholar and began working on a collaborative project trying to understand the behavior of an analytic object, called an L-series, and how the underlying algebraic function field determines the properties of the L-series. I spent a week in Chicago taking an introductory course in Maya, which is a program that creates interactive 3-dimensional environments that can be used in video games, animated films, and visual effects. Finally, I performed some consulting work with Jan Miller, who is the chair of the math department at Bay Mills Community College. In addition to assisting her with her required professional development, I was able to work with her in aligning BMCC curriculum with LSSU curriculum to assist their students with transferring mathematics courses from Bay Mills to Lake Superior State.

Accomplishments

The textbook [Algebraic Geometry: A Problem Solving Approach](#) was submitted to the American Mathematical Society for formal review. The authoring group has been in contact with the acquisitions editor throughout the writing of the text and we have received positive feedback before the formal submission. The text is undergoing editorial review by the AMS staff in conjunction with outside reviewers. The text was designed for inclusion in the Student Mathematical Library series and we expect to receive an update soon.

After completing the week-long course in Maya, I have been practicing the skills learned in the course to direct animated movies to help illustrate mathematical ideas. One such movie is designed to go with an exercise in the textbook. The problem requires the reader to construct a torus by puncturing two spheres and connecting the pieces with

rectangular sheets rolled up into a cylindrical shape. While the original solution consisted of a sequence of line diagrams, the solution I illustrated using Maya now consists of a 40 second animated movie with a soundtrack explaining the steps as they occur. This video was created after the text was submitted, but will be made available once the text is released.

Even though I will return to LSSU for the Spring 2012 semester, I have retained my status as a Visiting Scholar at Ohio State for the remainder of the academic year. This will afford me the opportunity to use the resources at OSU remotely. In addition to the traditional mathematical resources, I have also obtained access to the artistic databases for reference in my subsequent work examining the mathematical relationships in art and the artistic applications of mathematics.

Throughout the semester I met with Jan Miller, who is currently the chair of the mathematics department at Bay Mills Community College. As part of her professional development, she is required to complete coursework towards a master's degree in mathematics. Jan is fulfilling her requirements by taking coursework through the University of Houston. Due to the difficulty of the material and the impossibility of face-to-face guidance with the instructors, she met with me to assist her with the material, which consisted of coursework in both Abstract Algebra and Linear Algebra. In addition to working in mathematics, we worked together to align course objective and outcomes at BMCC to those at LSSU. Courses in algebra and statistics will have the same objectives at BMCC as the equivalent courses at LSSU.

Future Plans

Once the textbook has been published, the next goal of the authoring group is to offer workshops and seminars for people who wish to adopt the text. I have begun to create materials for use by any of my collaborators who wish to lead such a workshop. Material will consist of both print and multimedia to assist people who will teach using the textbook. I am working on examining L-series over algebraic function fields with finite characteristic in the hopes of understanding results published by Federico Pellarin this past October. With access to digital libraries, I hope to discover new examples for inclusion in a text detailing the interaction between art and mathematics. Finally, I hope to continue the collaboration and consultation between Bay Mills Community College and Lake Superior State University to our mutual benefit.